Basic Math Problem Solutions

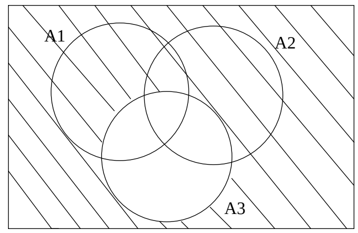
1) $A = \{2, 4, 6, 8, 10\}$, $B = \{1, 3, 5, 7, 9\}$, $C = \{1, 2, 3\}$, $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

a) $A \cup B = S$, b) $A \cup C = \{1, 2, 3, 4, 6, 8, 10\}$, c) $B \cap C = \{1, 3\}$

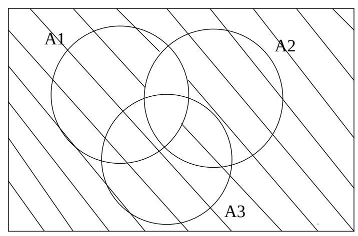
d) $(A \cap C)^c = \{1, 3, 4, 5, 6, 7, 8, 9, 10\}$, e) $B^c \cap C^c = \{4, 6, 8, 10\}$

2)

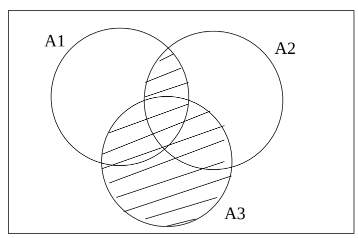
a) ${A_3}^c$



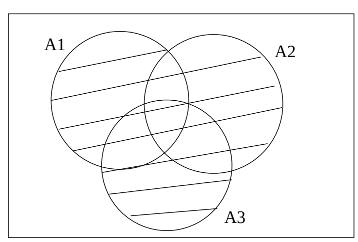
b) $(A_1 \cap A_2)^c$



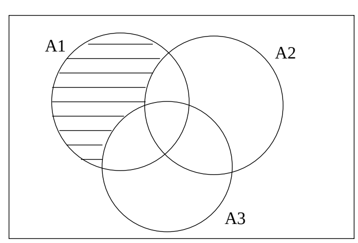
c) $(A_1 \cap A_2) \cup A_3$



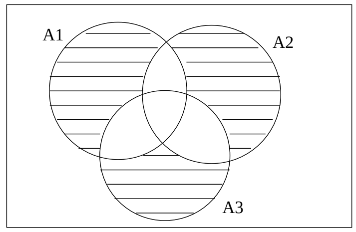
d) $A_1 \cup A_2 \cup A_3$



e) $A_1 \cap A_2^c \cap A_3^c$



f) $(A_1 \cap A_2^c \cap A_3^c) \cup (A_1^c \cap A_2 \cap A_3^c) \cup (A_1^c \cap A_2^c \cap A_3)$



3)

$$\begin{bmatrix} 4 & 3 & 0 & | & 2 \\ 0 & 3 & 4 & | & 4 \\ 1 & -1 & 1 & | & 0 \end{bmatrix} \overset{l_1 \leftrightarrow l_3}{\rightarrow} \begin{bmatrix} 1 & -1 & 1 & | & 0 \\ 0 & 3 & 4 & | & 4 \\ 4 & 3 & 0 & | & 2 \end{bmatrix} \overset{l_3 - 4l_1}{\rightarrow} \begin{bmatrix} 1 & -1 & 1 & | & 0 \\ 0 & 3 & 4 & | & 4 \\ 0 & 7 & -4 & | & 2 \end{bmatrix}$$

$$\overset{(1/3)l_2}{\rightarrow} \begin{bmatrix} 1 & -1 & 1 & | & 0 \\ 0 & 1 & 4/3 & | & 4/3 \\ 0 & 7 & -4 & | & 2 \end{bmatrix} \overset{l_1+l_2}{\rightarrow} \overset{l_3-7l_2}{\rightarrow} \begin{bmatrix} 1 & 0 & 7/3 & | & 4/3 \\ 0 & 1 & 4/3 & | & 4/3 \\ 0 & 0 & -40/3 & | & -22/3 \end{bmatrix}$$

$$\overset{(-3/40)l_3}{\rightarrow} \begin{bmatrix} 1 & 0 & 7/3 & | & 4/3 \\ 0 & 1 & 4/3 & | & 4/3 \\ 0 & 0 & 1 & | & 22/40 \end{bmatrix}$$

By back-substitution, we get $I_3 = 22/40 = \boxed{0.55}$, $I_2 = 4/3 - 4/3 \times 22/40 = \boxed{0.6}$, and $I_1 = 4/3 - 7/3 \times 22/40 = \boxed{0.05}$.

4) In the augmented matrix form, these equations become

$$\begin{bmatrix} 1 & 0 & 0 & -1 & | & 400 \\ 0 & 0 & 1 & -1 & | & 200 \\ 0 & 1 & -1 & 0 & | & 400 \\ 1 & -1 & 0 & 0 & | & -200 \end{bmatrix} \overset{l_4-l_1}{\rightarrow} \overset{l_2 \leftrightarrow l_3}{\rightarrow} \begin{bmatrix} 1 & 0 & 0 & -1 & | & 400 \\ 0 & 1 & -1 & 0 & | & 400 \\ 0 & 0 & 1 & -1 & | & 200 \\ 0 & -1 & 0 & 1 & | & -600 \end{bmatrix}$$

$$\overset{l_4+l_2}{\rightarrow} \begin{bmatrix} 1 & 0 & 0 & -1 & | & 400 \\ 0 & 1 & -1 & 0 & | & 400 \\ 0 & 0 & 1 & -1 & | & 200 \\ 0 & 0 & -1 & 1 & | & -200 \end{bmatrix} \overset{l_4+l_3}{\rightarrow} \begin{bmatrix} 1 & 0 & 0 & -1 & | & 400 \\ 0 & 1 & -1 & 0 & | & 400 \\ 0 & 0 & 1 & -1 & | & 200 \\ 0 & 0 & 0 & 0 & | & 0 \end{bmatrix}$$

$$\overset{l_2+l_3}{\rightarrow} \begin{bmatrix} 1 & 0 & 0 & -1 & | & 400 \\ 0 & 1 & 0 & -1 & | & 600 \\ 0 & 0 & 1 & -1 & | & 200 \\ 0 & 0 & 0 & 0 & | & 0 \end{bmatrix}$$

Now, if we let $x_4 = t$, then we must have $x_3 = 200 + t$, $x_2 = 600 + t$, and $x_1 = 400 + t$. There are infinitely many solutions to this system. Using the parameter t, they are represented by $\boxed{(400 + t, 600 + t, 200 + t, t)}$.

5) a)

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 2 \\ 2 & 1 & 4 & 3 \\ 3 & 1 & q & 6 \end{array}\right] \xrightarrow{l_2-2l_1} \xrightarrow{l_3-3l_1} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 2 \\ 0 & -1 & 0 & -1 \\ 0 & -2 & -6+q & 0 \end{array}\right]$$

$$\xrightarrow{(-1)l_2} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & -2 & -6+q & 0 \end{array}\right] \xrightarrow{l_1-l_2} \xrightarrow{l_3+2l_2} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & -6+q & 2 \end{array}\right]$$

If $q = 6$ then the system is not consistent. If $q \neq 6$ then there is a unique answer:

$$\xrightarrow{l_1-2l_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & (-10+q)/(-6+q) \\ 0 & 1 & 0 & 1 \\ 0 & 0 & -6+q & 2 \end{array}\right]$$

which gives us $\boxed{x_1 = (-10+q)/(-6+q),\ x_2 = 1,\ x_3 = 2/(-6+q)}$.

b)

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 2 & 1 & 1 & 3 \\ 1 & 4 & q & -2 \end{array}\right] \xrightarrow{l_2-2l_1} \xrightarrow{l_3-l_1} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 4 & -1+q & -4 \end{array}\right] \xrightarrow{l_3-4l_2} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 3+q & 0 \end{array}\right]$$

If $q \neq -3$ then we can continue the elimination to get a unique answer:

$$\xrightarrow{l_3/(3+q)} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 0 \end{array}\right] \xrightarrow{l_1-l_3} \xrightarrow{l_2+l_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

That is, $\boxed{x_1 = 2,\ x_2 = -1 \text{ and } x_3 = 0}$.
If $q = -3$ then the last equation disappears and we get: $\boxed{x_1 = 2 - t,\ x_2 = -1 + t,\ x_3 = t}$ where t is an arbitrary parameter. In this case, there are infinitely many solutions to the problem.

6) We are given the matrices

$$A = \begin{bmatrix} 3 & 0 & 1 \\ 2 & -1 & -2 \\ 1 & -7 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & 0 \\ -1 & 5 \\ 2 & 4 \end{bmatrix} \qquad C = \begin{bmatrix} -3 & 0 & 1 \\ 6 & -2 & 4 \end{bmatrix}$$

and asked to find, if possible, the quantities : AB, AC, $BB^T + 3A$, BA, $A(B^T + C)$ and $(B^T + C)A$. First of all, the matrix products, AC, BA and $A(B^T + C)$ are undefined because the row dimensions are not the same as the column dimensions. The remaining three results are;

$$AB = \begin{bmatrix} 14 & 4 \\ 5 & -13 \\ 17 & -23 \end{bmatrix} \qquad BB^T = \begin{bmatrix} 25 & -4 & 11 \\ 2 & 23 & 12 \\ 11 & -3 & 29 \end{bmatrix}$$

and

$$(B^T + C)A = \begin{bmatrix} 4 & -20 & 12 \\ 32 & -59 & 24 \end{bmatrix}$$