Basic Probability and Statistics Problem Answers

1) (a) 118,813,760 (b) 59,406,880 (c) 97,656,250
2) ≈ 0.976
3) ≈ 0.51
4) (a) 0.76 (b) 0.04 (c) 0.19 (d) 0.01 (e) 0.80
5) (a) 0.50 (b) 0.35 (c) 0.5 (d) $0.33\dot{3}$ (e) 0.4118
6) (a) 0.03 (b) 0.05 (c) 0.02 (d) 0.07 (e) 0.88 (f) 0.118
   g) (i) 0.254 (ii) 0.136 (iii) 0.390
7) 0.64
8) (a) 0.2865 (b) 0.121
9) (a) 0.0475 (b) 849,500
10) a) One must assume that the lengths of the spanner bushings are normally distributed. When one looks at a histogram of the sampled data, this does not appear to be the case. Granted the sample size is small, but the validity of the assumption is suspect.
    b) $[1.137, 1.139]$
    c) The supply shop's claim appears to be suspect.
    d) $[2 \times 10^{-3}, 4.9 \times 10^{-3}]$ inches
    e) The collected data is paired. Thus, if one wishes to compare the students' average measurements one must look at the *differences* among the individual measurements.
    f) One must assume that the population of differences is normally distributed. When one looks at histogram of the sampled differences, this does not appear to be the case. However, this is a very small sample and it is difficult to say.
    g) $[-3.4 \times 10^{-4}, 1.5 \times 10^{-4}]$ inches.
    h) There is no evidence to show that the students are not coming up with the same mean measurement.
    i) Since these are **not independent** samples.
11) a) $[0.527, 0.649]$
    b) $[0.022, 0.074]$
    c) $[0.47, 0.61]$
    d) It is unlikely that the two speeds of operation yield the same probability of obtaining a faulty bag.
12) a) $[0.16, 1.66]$ inches^2
    b) Based on (a) (since $1 \in [0.16, 1.66]$), the assumption of equal variances appears reasonable. When one looks at histograms of the sampled data, the assumption of normally distributed populations seems reasonable as well.
    c) $[7.2 \times 10^{-5}, 5.48 \times 10^{-4}]$
    d) There may be evidence that the hourly mean pitch diameter may be unequal.
13) a) $[1.4 \times 10^{-2}, 3.3 \times 10^{-2}]$
    b) The population of wear in coated valves must be normally distributed. When one looks at a histogram of the sampled data, this assumption seems suspect as the distribution looks skewed.
14) a) $H_o: \ p = 0.25$ (or $p \geq 0.25$; taking $p = 0.25$ is the worst case)
    $H_a: \ p < 0.25$
    b) Type I error is rejecting H_o when it is true. For the marketing director, it means concluding that the market share is less than 25%, when in fact it isn't.
    c) Type II error is not rejecting H_o when it is false. For the marketing director, it means concluding that the market share is 25%, when in fact it isn't.
    d) One could argue both ways. Conceivably, type II error is more serious.

15) a) We must reject H_o at the α = 0.05 significance level and conclude that the mean measured length is not 1.14 in. The exact p-value is 0.0078.

b) We cannot reject H_o at the α = 0.05 significance level and conclude that there is insufficient evidence to say that the two students' average measurements are not the same. The exact p-value is 0.4228.

c) Yes, the same conclusion could have been drawn based on our previously build 95% C.I..

16) a) We must reject H_o at the α = 0.05 significance level and conclude that the probability of obtaining a faulty bag under high-speed operation is more than 50%. The p-value is $P(Z > 2.78) = 0.0027$.

b) We cannot reject H_o at the α = 0.05 significance level and conclude that there is insufficient evidence to show that the probability of obtaining a faulty bag under low-speed operation is more than 4%. The p-value is $P(Z > .645) = 0.2595$.

c) We must reject H_o at the α = 0.05 significance level and conclude that the probabilities of obtaining a faulty bag are not the same at the two different speeds of operation. The p-value is ≈ 0.

17) We must reject H_o at the α = 0.05 significance level and conclude that there is an hourly difference in the pitch diameter. The p-value is $2P(T > 2.888) = 2(0.0049) = 0.0098$.