

### Tutorial Problems for Chapters 1 and 2

- 1) Consider the sample space

$$S = \{\text{copper, sodium, nitrogen, potassium, uranium, oxygen, zinc}\}$$

and the events

$$A = \{\text{copper, sodium, zinc}\}$$

$$B = \{\text{sodium, nitrogen, potassium}\}$$

$$C = \{\text{oxygen}\}$$

List the elements of the sets corresponding to the following events;

- a)  $A^c$ ,
  - b)  $A \cup C$ ,
  - c)  $(A \cap B^c) \cup C^c$ ,
  - d)  $B^c \cap C^c$ ,
  - e)  $A \cap B \cap C$ ,
  - f)  $(A^c \cup B^c) \cap (A^c \cap C)$ .
- 2) Let  $A$ ,  $B$ , and  $C$  be events in the sample space  $S$ . Using Venn diagrams, shade the areas representing the following events;
- a)  $(A \cap B)^c$ ,
  - b)  $(A \cup B)^c$ ,
  - c)  $(A \cap C) \cup B$ .
- 3) In how many different ways can a true-false test consisting of 9 questions be answered,
- a) if an answer is given to each question?
  - b) if not answering is an option? (Some true-false tests are such that you get one point for getting it right, and lose two points for getting it wrong. If you are unsure, it is thus better not to answer.)
- 4) If a multiple-choice test consists of 5 questions, each with 4 possible answers, of which only one is correct,
- a) In how many ways can a student check off one answer to each question?
  - b) In how many ways can a student check off one answer to each question and get all the answers wrong?
- 5)
- a) In how many ways can 6 people be lined up to get on a bus?
  - b) If 3 specific persons insist on following each other, how many ways are possible?
  - c) If 2 specific persons refuse to follow each other, how many ways are possible?
- 6) Four married couples have bought 8 seats in the same row for a concert. In how many ways can they be seated,
- a) with no restrictions?
  - b) if each couple is to sit together?
  - c) if all men sit together to the right of all the women (assuming traditional marriages)?
- 7) How many ways are there to select 3 candidates from 8 equally qualified recent graduates for openings in an engineering firm?
- 8) Suppose that in a senior college class of 500 students, it is found that 210 smoke, 258 drink alcoholic beverages, 216 eat between meals, 122 smoke and drink alcoholic beverages, 83 eat between meals and drink alcoholic beverages, 97 smoke and eat between meals, and 52 engage in all three of these bad health practices. If a member of this senior class is selected at random, find the probability that the student
- a) smokes but does not drink alcoholic beverages;
  - b) eats between means and drinks alcoholic beverages but does not smoke;
  - c) neither smokes nor eats between meals.
- 9) The probability that an American industry will locate in Munich is 0.7, the probability that it will locate in Brussels is 0.4, and the probability that it will locate in either Munich or Brussels or both is 0.8. What is the probability that the industry will locate
- a) in both cities?
  - b) in neither city?
- 10) A random sample of 200 adults are classified below by sex and their level of education attained;

Education	Male	Female
Elementary	38	45
Secondary	28	50
University	22	17

If a person is picked at random from this group, find the probability that

- a) the person is male, given that the person has a secondary education;
- b) the person does not have a university degree, given that the person is female.

- 11) In the senior year of a high school graduating class of 100 students, 42 studied mathematics, 68 studied psychology, 54 studied history, 22 studied both mathematics and history, 25 studied both mathematics and psychology, 7 studied history but neither mathematics nor psychology, 10 studied all three subjects, and 8 did not take any of the three. If a student is selected at random, find the probability that
- a person enrolled in psychology takes all three subjects;
  - a person not taking psychology is taking both history and mathematics.
- 12) The probability that a doctor correctly diagnoses a particular illness is 0.7. Given that the doctor makes an incorrect diagnosis, the probability that the patient enters a lawsuit is 0.9. What is the probability that the doctor makes an incorrect diagnosis and the patient sues?
- 13) A town as 2 fire engines operating independently. The probability that a specific engine is available when needed is 0.96.
- What is the probability that neither is available when needed?
  - What is the probability that a fire engine is available when needed?
- 14) In a certain region of the country it is known from past experience that the probability of selecting an adult over 40 years of age with cancer is 0.05. If the probability of a doctor correctly diagnosing a person with cancer as having the disease is 0.78 and the probability of incorrectly diagnosing a person without cancer as having the disease is 0.06, what is the probability that a person is diagnosed as having cancer?
- 15) Classify the following random variables as discrete or continuous:
- $X$ : the number of automobile accidents per year in Nova Scotia.  
 $Y$ : the length of time to play 18 holes of golf.  
 $M$ : the amount of milk produced yearly by a particular cow.  
 $N$ : the number of eggs laid each month by a hen.  
 $P$ : the number of building permits issued each month in Halifax.  
 $Q$ : the weight of grain produced per acre on a farm.
- 16) Determine the value  $c$  so that each of the following functions become valid probability mass functions for the discrete random variable  $X$ ;
- $f(x) = c(x^2 + 4)$  for  $x = 0, 1, 2$ , and 3.
  - $f(x) = c \binom{2}{x} \binom{3}{3-x}$  for  $x = 0, 1$ , and 2.
- 17) The total number of hours, measured in units of 100 hours, that a family runs a vacuum cleaner over a period of one year is a continuous random variable,  $X$ , that has the probability density function

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 \leq x < 2 \\ 0, & \text{elsewhere} \end{cases}$$

Find the probability that over a period of one year, a family runs their vacuum cleaner

- less than 120 hours;
- between 50 and 100 hours.

- 18) A shipment of 7 television sets contains 2 defective sets. A hotel makes a random purchase of 3 of the sets. If  $X$  is the number of defective sets purchased by the hotel, find the probability mass function of  $X$ .
- 19) The probability mass function of  $X$ , the number of imperfections per 10 metres of a synthetic fabric in continuous rolls of uniform width, is given by

$x$	0	1	2	3	4
$f(x)$	0.41	0.37	0.16	0.05	0.01

Construct the cumulative distribution function of  $X$ .

- 20) Consider the probability density function

$$f(x) = \begin{cases} k\sqrt{x}, & \text{for } 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

- Evaluate the required value of  $k$ .
  - Find  $F(x)$  and use it to evaluate  $P[0.3 < X \leq 0.6]$ .
- 21) The probability mass function of the discrete random variable  $X$  is

$$f(x) = \binom{3}{x} (0.25)^x (0.75)^{3-x}, \quad \text{for } x = 0, 1, 2, 3.$$

Find the mean of  $X$ .

- 22) Recall question #17 above. Find the mean number of hours per year that families run their vacuum cleaners.  
 23) Let  $X$  be a random variable with the following probability mass function

$x$	-3	6	9
$f(x)$	1/6	1/2	1/3

Find the expected value of  $g(X) = (2X + 1)^2$ .

- 24) Recall question #17 above. Find the variance of the number of hours per year that families run their vacuum cleaners.  
 25) Let  $X$  be a random variable with the following probability distribution function

$x$	-3	6	9
$f(x)$	1/6	1/2	1/3

Find  $E[X]$  and  $E[X^2]$  and then, using these values, re-evaluate  $E[(2X + 1)^2]$ .

- 26) If a random variable  $X$  is defined such that

$$E[(X - 1)^2] = 10, \quad E[(X - 2)^2] = 6$$

then find its mean and variance,  $\mu$  and  $\sigma^2$ .

- 27) According to *Chemical Engineering Progress* (Nov. 1999) approximately 30% of all pipework failures in chemical plants are caused by operator error.  
 a) What is the probability that no more than 4 out of the next 20 such failures are due to operator error?  
 b) Suppose, for a particular plant, that, out of a random sample of 20 such failures, exactly 5 are due to operational errors. Does this cast doubt on the 30% figure stated above for this plant? Comment.
- 28) Suppose that airplane engines operate independently and fail with probability 0.4. Assuming that a plane makes a safe flight if at least one-half of its engines run, determine whether a 4-engine plane or a 2-engine plane is safer.
- 29) The probability that a student pilot passes the written test for a private pilot's license is 0.7. Find the probability that the student will pass the test  
 a) on the third try;  
 b) before the fourth try.
- 30) The number of failures of a testing instrument from contamination particles on the product is a Poisson distributed random variable with a mean of 0.02 failures per hour.  
 a) What is the probability that the instrument does not fail in an 8-hour shift?  
 b) What is the probability of at least one failure in a 24-hour day?
- 31) A soft-drink machine is regulated so that it discharges an average of 200 millilitres per cup. If the amount of drink is normally distributed with a standard deviation equal to 15 millilitres,  
 a) what fraction of the cups will contain more than 224 millilitres?  
 b) what is the probability that a cup contains between 191 and 209 millilitres?  
 c) how many cups will probably overflow if 230 millilitre cups are used for the next 1000 drinks?  
 d) below what value do we get the smallest 25% of the drinks?
- 32) In a certain city, the daily consumption of electric power, in millions of kilowatt-hours, is a random variable  $X$  having a Gamma distribution with a mean of 6 and a variance of 12.  
 a) Find the values of the parameters of the Gamma distribution.  
 b) Find the probability that on any given day the daily power consumption will exceed 12 million kilowatt-hours.
- 33) The life, in years, of a certain type of electrical switch has an exponential distribution with an average life of 2 years. If 100 of these switches are installed in different systems, what is the probability that between 35 and 40 (inclusive) fail during the first year?
- 34) Computer response time is an important application of the Gamma and exponential distributions. Suppose that a study of a certain computer system reveals that the response time in seconds has an exponential distribution with a mean of 3 seconds.  
 a) What is the probability that the response time exceeds 5 seconds?  
 b) Given that the computer has not responded for 5 seconds, what is the probability that it takes at least 5 more seconds to respond?
- 35) The amount of time that a drive-through bank teller spends on a customer is a random variable with a mean of 3.2 minutes and a standard deviation of 1.6 minutes. If a random sample of 64 customers is observed, find the probability that their sample mean time at the teller is  
 a) at most 2.7 minutes;  
 b) more than 3.5 minutes;  
 c) at least 3.2 minutes but less than 3.4 minutes

- 36) A taxi company is trying to decide whether to purchase brand A or brand B tires for its fleet of taxis. To estimate the difference in the two brands, an experiment is conducted using 30 of each brand. Sixty randomly chosen taxis participated in the experiment. The tires are run until they wear out. The results are as follows: Brand A tires had a sample mean life of 36,300 km and a sample standard deviation of 6000 km; Brand B tires had a sample mean life of 38,100 km and a sample standard deviation of 7200 km. Compute a 95% confidence interval for the difference in the two population means. Make and state any necessary assumptions.
- 37) Reconsider question #36. Suppose the taxi company had proceeded by assigning a tire of each brand to the rear wheels of each of 30 randomly selected taxis and the following distances (in km) were recorded:

Brand A	34,400	45,500	36,700	32,000	48,400	32,800	38,100	30,100	39,000	31,100
Brand B	36,700	46,800	37,700	31,100	47,800	36,400	38,900	31,500	37,800	34,900
Brand A	35,400	31,900	40,000	33,900	37,900	32,200	34,300	38,000	34,700	43,000
Brand B	37,700	33,900	37,500	35,800	39,000	35,900	35,900	37,900	37,000	41,000
Brand A	36,300	31,300	33,000	33,600	35,900	34,200	32,400	38,400	32,300	33,500
Brand B	35,400	36,100	37,100	34,600	35,000	35,500	35,500	41,900	35,200	31,000

How do the two brands compare now?

- 38) A new rocket-launching system is being considered for deployment of small, short-range rockets. The existing system has  $p = 0.8$  as the probability of a successful launch. A sample of 40 experimental launches is made with the new system and 34 are successful.
- Construct a 95% confidence interval for  $p$ .
  - Would you conclude that the new system is better?
- 39) A study is to be made to estimate the percentage of citizens in a town who favour having their water fluoridated. How large a sample is needed if one wishes to be at least 95% confident that our estimate is within 1% of the true percentage?
- 40) A clinical trial is conducted to determine if a certain type of inoculation has an effect on the incidence of a certain disease. A sample of 1000 rats was kept in a controlled environment for a period of 1 year and 500 rats were given the inoculation. Of the group not given the drug, there were 120 incidences of the disease, while 98 of the inoculated group contracted it. If we call  $p_1$  the probability of incidence of the disease in the uninoculated rats and  $p_2$  the probability of incidence in the inoculated rats, compute a 90% confidence interval for  $p_1 - p_2$ . What is this telling you?
- 41) A large manufacturing firm is being charged with discrimination in its hiring practices.
- What hypothesis is being tested if a jury commits a type I error by finding the firm guilty?
  - What hypothesis is being tested if a jury commits a type II error by finding the firm guilty?
- 42) The proportion of adults living in a small town who are university graduates is estimated to be  $p = 0.3$ . To test this hypothesis, a random sample of 15 adults is selected. If the number of university graduates in our sample is anywhere from 2 to 7, we shall not reject the null hypothesis; otherwise, we shall conclude that  $p \neq 0.3$ .
- What is the probability,  $\alpha$ , of a type I error?
  - What is the probability,  $\beta$ , of a type II error if the true proportion is  $p = 0.2$ ?
  - Is this a good test procedure?
- 43) A manufacturer has developed a new fishing line, which she claims has a mean breaking strength of 15 kg with a standard deviation of 0.5 kg. To test the hypothesis that  $\mu = 15$  kg against the alternative that  $\mu < 15$  kg, a random sample of 50 lines will be tested. The critical region is defined to be  $\bar{x} < 14.9$ . Assume that the standard deviation is known to be  $\sigma = 0.5$ .
- Find the probability of committing a type I error.
  - Find the probability of committing type II errors for the alternatives  $\mu = 14.9$  and  $\mu = 14.8$  kg.
- 44) It is claimed that an automobile is driven, on average, more than 20,000 km per year. To test this claim, a random sample of 100 automobile owners are asked to keep a record of the kilometres they travel. Would you agree with this claim if the random sample showed a sample mean of 23,500 km and a sample standard deviation of 3900 km? Use a p-value in your conclusion.
- 45) In a study to estimate the proportion of residents in a certain city and its suburbs who favour the construction of a nuclear power plant, it is found that 63 of 100 urban residents favour the construction while only 59 of 125 suburban residents are in favour. Is there a significant difference between the proportion of urban and suburban residents who favour construction of the nuclear power plant? Use a p-value in your conclusion.